THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010 I/J University Mathematics 2015-2016
Problem Set 3

1. Evaluate each of the following limits.

   (a) $\lim_{x\to 2} \frac{2-x}{3-\sqrt{x^2+5}}$

   (b) $\lim_{x\to \pi} \frac{\sin x}{\pi - x}$

   (c) $\lim_{x\to 0} \frac{\sin 6x}{\sin 5x}$

   (d) $\lim_{x\to 0} \frac{1-2\cos x+\cos 2x}{x^2}$;

   (e) $\lim_{x\to +\infty} \sqrt{x^2+x}-x$

   (f) $\lim_{x\to +\infty} x(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x)$

2. Evaluate each of the following limits.

   (a) $\lim_{x\to 0} \frac{2^x-2^{-x}}{2^x+2^{-x}}$

   (b) $\lim_{x\to +\infty} \frac{2^x-2^{-x}}{2^x+2^{-x}}$

   (c) $\lim_{x\to -\infty} \frac{2^x-2^{-x}}{2^x+2^{-x}}$

3. Evaluate each of the following limits.

   (a) $\lim_{x\to 0} \frac{\sin 2x}{x}$

   (b) $\lim_{x\to +\infty} \frac{\sin 2x}{x}$

4. Let $f(x) = |x+1| + |x-1|$

   (a) Rewrite $f(x)$ as a piecewise defined function by filling the following blanks:

$$f(x) = \begin{cases} \underline{\hspace{3cm}} & \text{if} \quad x \geq 1; \\ \underline{\hspace{3cm}} & \text{if} \quad -1 \leq x < 1; \\ \underline{\hspace{3cm}} & \text{if} \quad x \leq -1. \end{cases}$$

   (b) Find $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1^-} f(x)$. Does $\lim_{x\to 1} f(x)$ exist?

5. Let a be a real number and let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} e^{\frac{1}{x}} & \text{if} \quad x < 0; \\ 2 & \text{if} \quad x = 0; \\ a\cos x & \text{if} \quad x > 0 \end{cases}$$

   If $\lim_{x\to 0} f(x)$ exists, find the value of a.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} 1 & \text{if } x = \dfrac{1}{n} \text{ for some } n \in \mathbb{N}; \\ \\ 0 & \text{otherwise.} \end{cases}$$

(a) Prove that $\lim_{x \to 0} f(x)$ does not exist.

(b) Prove that $\lim_{x \to \frac{1}{3}} f(x) = 0$.